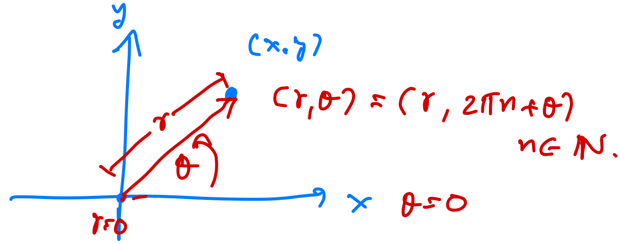
ทบทวน: ภาพใน 2 มิติ.

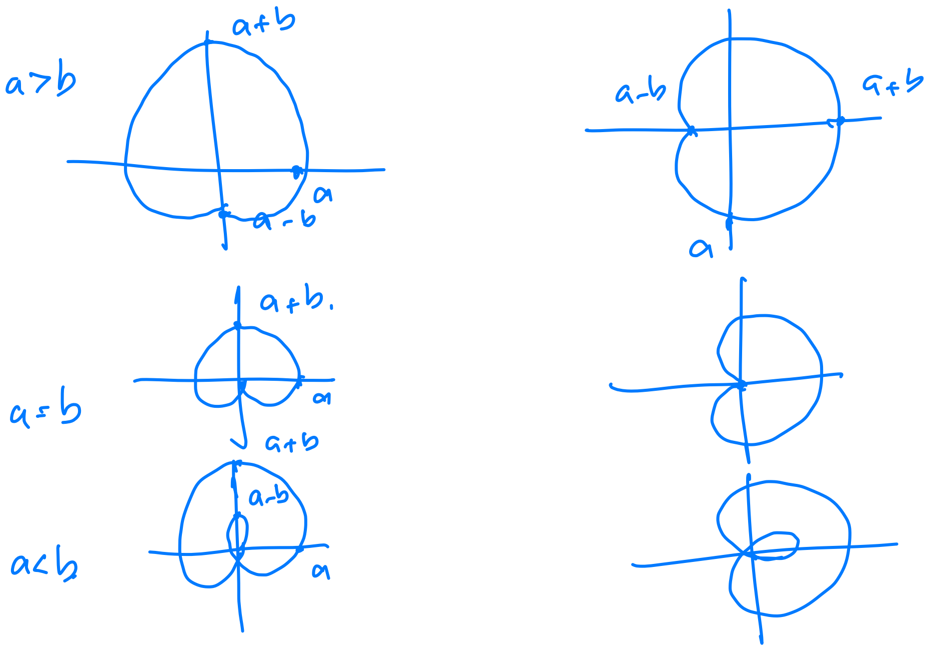
1. พิกัดฉาก. (x, y)
2. พิกัดเชิงขั้ว. (r, θ)



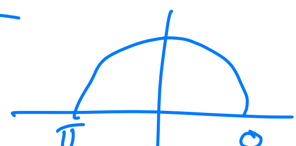
ภาพลีมาซอง

- $r = a \pm b \sin\theta$

$r = a \pm b \cos\theta$

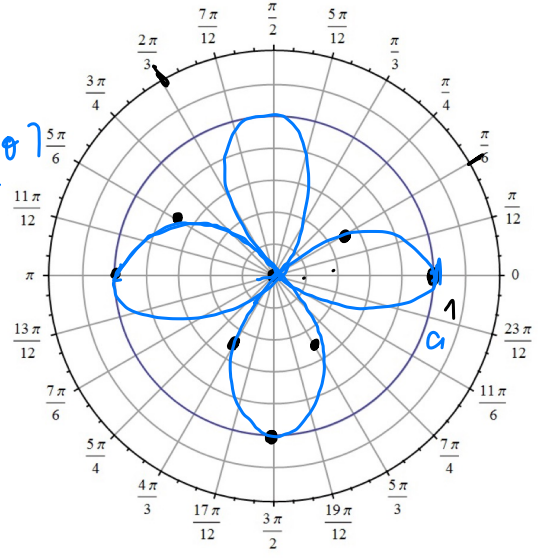


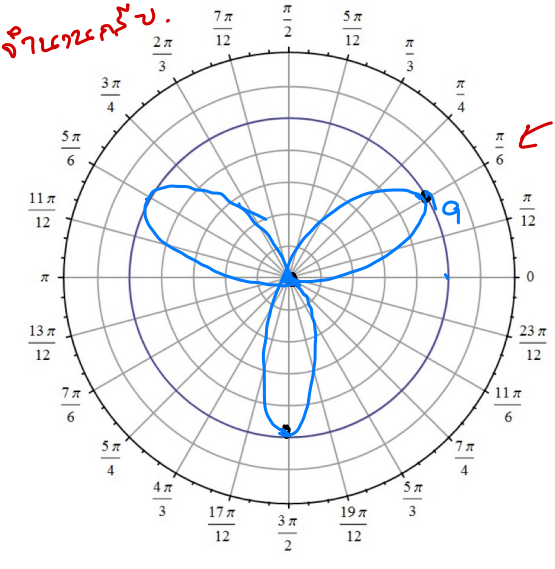
=> เริ่มโค้งกลีบกุหลาบ.



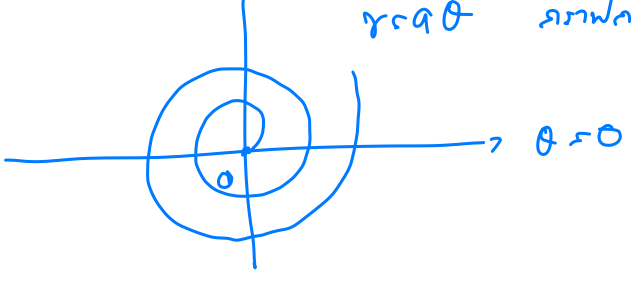
Ex: $r = \cos(2\theta)$ — [illegible]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$r = \cos(2\theta)$	1	0.5	0	-0.5	-1	-0.5	0.	0.5	1





$r = a\theta$.



ตัวอย่าง 4.2.11 จงจับคู่กราฟของเส้นโค้งในพิกัดเชิงขั้วต่อไปนี้กับสมการที่กำหนดให้

(1) $r = 2 - \sin 2\theta$ (2) $r = 3\cos 2\theta$ (3) $r = \theta + 1$

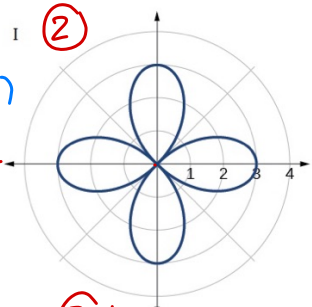
(4) $r^2 = -4\cos 2\theta$ (5) $r = 1 + 3\sin\theta$ (6) $r = 5 + 7\sin\theta$

(7) $r = 7 + 4\sin\theta$ (8) $r = 5 + 4\cos\theta$ (9) $r = 2\sqrt{1 - \sin^2\theta}$

$r^2 = 4\cos^2\theta$, $= 2\cos\theta$

$\cos^2\theta$

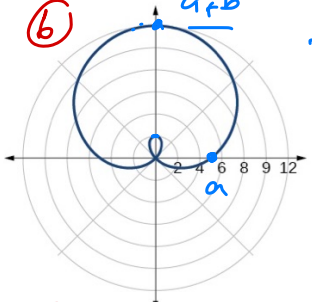
I (2)



$r = a\cos(n\theta)$

$a = 3$, $n = 2$

II (6)



$a+b$

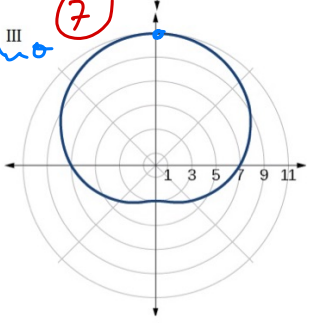
$r = a + b\cos\theta$

$a < b$

$a + b = 12$

$a - b = -2$

III (7)

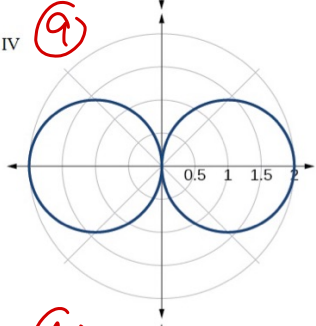


$r = a + b\cos\theta$

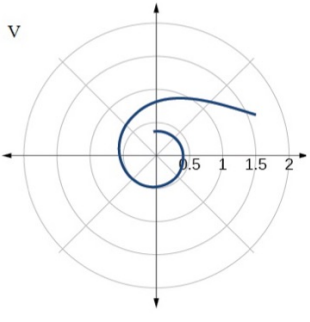
$a > b$

$a + b = 11$

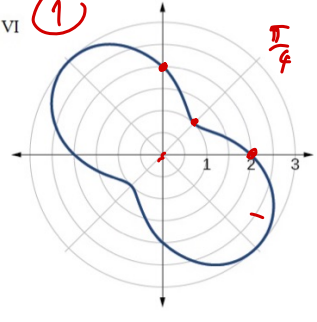
IV (9)



V (8)



VI (1)

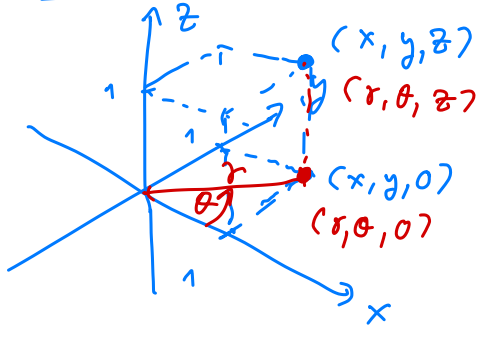


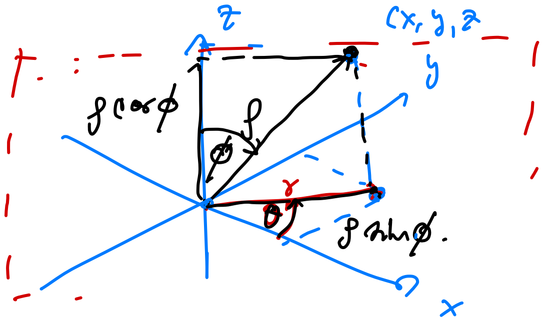
$\frac{\pi}{4}$

$r = 2 - \sin 2\theta$

ระบบพิกัดฉาก 3 มิติ:

- พิกัดฉาก (x, y, z)
- พิกัดทรงกระบอก (r, θ, z)
- พิกัดทรงกลม (ρ, ϕ, θ)



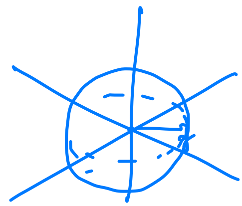


$$\rho = \sqrt{x^2+y^2+z^2}$$

ความสัมพันธ์:

พิกัดฉาก (x, y, z)	พิกัดทรงกระบอก (r, θ, z)	พิกัดทรงกลม (ρ, ϕ, θ)
x	$x = r\cos\theta$	$x = \underbrace{\rho\sin\phi}_{r}\cos\theta$
y	$y = r\sin\theta$	$y = \underbrace{\rho\sin\phi}_{r}\sin\theta$
z	$z = z$	$z = \rho\cos\phi$.

⇒ ตัวอย่างกราฟสำคัญใน 3 มิติ.



① . ทรงกลม: $x^2+y^2+z^2 = R^2$

พิกัดฉาก.

พิกัดทรงกระบอก:

$$(r\cos\theta)^2 + (r\sin\theta)^2 + z^2 = R^2$$

$$r^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} + z^2 = R^2$$

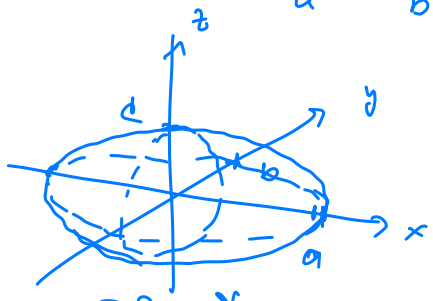
พิกัดทรงกลม:

$$(\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2 + (\rho\cos\theta)^2 = R^2$$

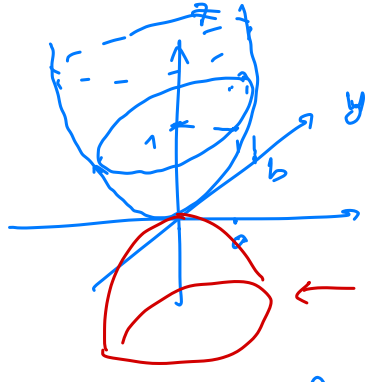
$$\rho^2\sin^2\phi\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} + \rho^2\cos^2\theta = R^2$$

$$\rho^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} = R^2 \Rightarrow \rho = R$$

② ทรงรี: ผิวของ ทร. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

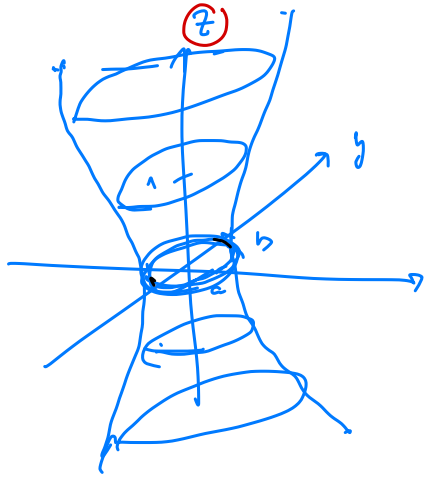


③: พาราโบลอยด์เชิงวงรี: $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

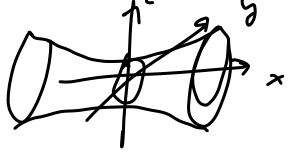


$\leftarrow z=-\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)$

④. ไฮเพอร์โบลอยด์เชิงเดียว: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

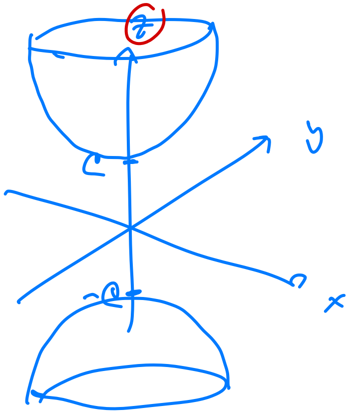


$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

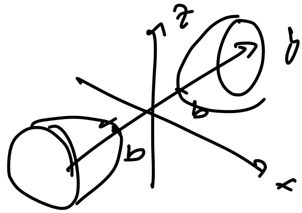


5. ໄຮເປີໂບລອຍ ແຍກສ່ວນ:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

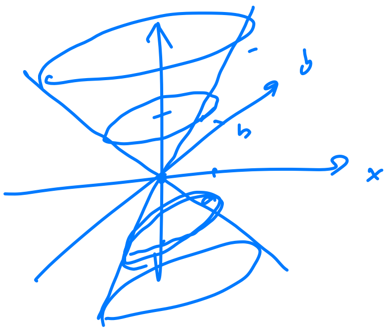


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$$

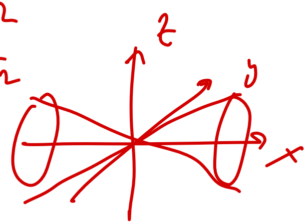


6. ກວຍເອລີບ:

$$z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2 = 0$$



$$x^2 = \frac{y^2}{b^2} + \frac{z^2}{c^2}$$



ตัวอย่าง 4.3.10 จงจับคู่กราฟของพื้นผิวในสามมิติต่อไปนี้กับสมการที่กำหนดให้

(1) $x^2 + y^2 + 4z^2 = 10$ (2) $z^2 + 4y^2 - 4x^2 = 4$ (3) $9y^2 + z^2 = 16$

(4) $y^2 + z^2 = x^2$ (5) $z = -4x^2 - y^2$ (6) $x = -y^2 - z^2$

(7) $x^2 + 2z^2 = 8$ (8) $z^2 + x^2 - y^2 = 1$ (9) $x^2 + 4z^2 = y^2$

(10) $9x^2 + 4y^2 + 2z^2 = 36$