An ADI Crank-Nicolson Orthogonal Spline Collocation Method for the Two-Dimensional Fractional Diffusion-Wave Equation David BECKHAM[1]

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Abstract: A new method is formulated and analyzed for the approximate solution of a two-dimensional time-fractional diffusion-wave equation. In this method, orthogonal spline collocation (OSC) is used for the spatial discretization and, for the time-stepping, a novel alternating direction implicit (ADI) method based on the Crank-Nicolson method combined with the L1-approximation of the time Caputo derivative. OSC has evolved as a valuable technique for the solution of several types of partial differential equations. Their popularity is due in part to their conceptual simplicity, wide applicability and ease of implementation. Since their introduction over fifty years ago, ADI methods have been employed effectively for the time stepping in the numerical solution of a variety of time-dependent multidimensional problems. Their primary attraction is that they reduce such a problem to independent systems of one-dimensional problems. In this presentation, we prove that the new scheme is stable, and of optimal accuracy in various norms.

Keywords: Two-dimensional fractional diffusion-wave equation, Caputo derivative, alternating direction implicit method, optimal global error estimates, superconvergence.

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